Autumn 2025
Professor Sowinski

Physics 141: Exam IV

University of Rochester
Department of Physics & Astronomy

Instructions

There is a short answer section, and two long answer ones.
You have 100 min to catch up to 33 ☄'s. That's 3 min per ☄.
No partial stars can be earned on this exam so write understandably.
Draw arrows to let me know the direction of your explanations.
Accompany them with labeled and neat diagrams.
Write legibly with a font size greater than 12.
May the Force be with you.

On my honor and reputation as a scholar of Natural Science,
I will neither give nor receive unauthorized help on this exam,
and hereby vow that all work will be my own.

Signature/Print ____

Short	Long
☄☄	☄☄☄
☄☄	☄
☄☄	☄☄☄
☄☄	☄☄
☄☄	☄
☄☄	☄
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☄☄	☄☄
☄☄	

$G = 6.674 \times 10^{-11} \dfrac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$	$c = 2.998 \times 10^8 \dfrac{\text{m}}{\text{s}}$	$h = 6.626 \times 10^{-34} \text{J} \cdot \text{s}$	$k_B = 1.381 \times 10^{-23} \dfrac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K}}$
$M_\oplus = 5.972 \times 10^{24}\text{kg}$	$M_\odot = 1.989 \times 10^{30}\text{kg}$	$R_\oplus = 6.371 \times 10^6\text{m}$	$R_\odot = 6.957 \times 10^8\text{m}$
$AU = 1.496 \times 10^{11}\text{m}$	$M_☾ = 7.348 \times 10^{22}\text{kg}$	$R_☾ = 1737\text{km}$	$r_{☾-\oplus} = 3.844 \times 10^5\text{km}$

	Cartesian Coordinates	Polar Coordinates	Cartesian	$\Leftrightarrow$ Polar
	$\boldsymbol{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$	$\boldsymbol{r} = r\hat{\boldsymbol{r}}$	$x = r\cos\theta$	$r = \sqrt{x^2+y^2}$
$\boldsymbol{v} = \dfrac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}}$	$\dot{\boldsymbol{r}} = \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}}$	$\dot{\boldsymbol{r}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$	$y = r\sin\theta$	$\theta = \arctan\frac{y}{x}$
$\boldsymbol{a} = \dfrac{d\boldsymbol{v}}{dt} = \dot{\boldsymbol{v}}$	$\ddot{\boldsymbol{r}} = \ddot{x}\hat{\boldsymbol{\imath}} + \ddot{y}\hat{\boldsymbol{\jmath}}$	$\ddot{\boldsymbol{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$		

Inertial Frame	Rotating Frame
$\boldsymbol{v} = \dfrac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}}$	$\boldsymbol{v} = \boldsymbol{v}_{\text{rot}} + \boldsymbol{\Omega} \times \boldsymbol{r}$
$\boldsymbol{a} = \dfrac{d\boldsymbol{v}}{dt} = \dot{\boldsymbol{v}}$	$\boldsymbol{a} = \boldsymbol{a}_{\text{rot}} + 2\boldsymbol{\Omega} \times \boldsymbol{v}_{\text{rot}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r})$

Newton's 2nd Law :	$\boldsymbol{F} = \dfrac{d\boldsymbol{p}}{dt}$	Momentum :	$\boldsymbol{p} = m\boldsymbol{v}$
Impulse :	$\Delta\boldsymbol{p} = \int \boldsymbol{F}dt$	Universal Gravitation :	$\boldsymbol{F}_g = -G\dfrac{m_1 m_2}{\lvert\boldsymbol{r}_2 - \boldsymbol{r}_1\rvert^3}(\boldsymbol{r}_2 - \boldsymbol{r}_1)$
Work :	$W = \int_C \boldsymbol{F}(\boldsymbol{r}) \cdot d\boldsymbol{r}$	Kinetic Energy:	$K = \dfrac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
Work-Energy Theorem:	$W = \Delta K$	Conservative Force :	$\boldsymbol{F} = -\nabla U$
Spring Potential Energy :	$U_s = \dfrac{1}{2}m\omega^2\lvert\boldsymbol{r} - \boldsymbol{r}_{eq}\rvert^2$	Potential Energy :	$U(\boldsymbol{r}_f) - U(\boldsymbol{r}_i) = -\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$
Grav. Potential Energy :	$U_g = -G\dfrac{m_1 m_2}{\lvert\boldsymbol{r}_2 - \boldsymbol{r}_1\rvert}$	Mass of object :	$M = \int_\Omega \rho(\boldsymbol{r})dV$
Center of mass:	$\boldsymbol{R} = \dfrac{1}{M}\sum_i m_i\boldsymbol{r}_i$ or $\dfrac{1}{M}\int_\Omega \rho(\boldsymbol{r})\boldsymbol{r}dV$		
Angular Momentum:	$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = I\boldsymbol{\omega}$	Torque:	$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = I\boldsymbol{\alpha}$
Moment of Inertia:	$I = \sum_i m_i(r_i^\perp)^2$		

$$\boldsymbol{A} \cdot \boldsymbol{B} = \lvert\boldsymbol{A}\rvert\lvert\boldsymbol{B}\rvert\cos\theta \quad \lvert\boldsymbol{A} \times \boldsymbol{B}\rvert = \lvert\boldsymbol{A}\rvert\lvert\boldsymbol{B}\rvert\sin\theta \quad \boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B})$$

Expanding δx around x $\quad f(x + \delta x) = f(x) + f'(x)\delta x + \dfrac{1}{2!}f''(x)\delta x^2 + \cdots$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - \cdots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \qquad \ln\lvert 1 + x\rvert = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots \qquad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \qquad \frac{1}{1 - x} = 1 + x + x^2 + x^3 + \cdots$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\beta\sin\alpha$$

$$\sin 2\theta = 2\sin\theta\cos\theta \quad \cos 2\theta = \cos^2\theta - \sin^2\theta \qquad \cos\tfrac{\theta}{2} = \sqrt{\tfrac{1+\cos\theta}{2}} \quad \sin\tfrac{\theta}{2} = \sqrt{\tfrac{1-\cos\theta}{2}}$$

Short Answer Questions

Responses are to be qualitative, no more than 3-4 sentences long. Pictures can accompany explanations; keep equations to a minimum. Each of these questions is from one of the sections in K&K and will test how well you read the book.

(💣) Write Kepler's Three Laws. (💣) What property of Universal Gravitation ensures that they are satisfied?

(💣💣) Describe all possible orbits ($L \neq 0$) of two point masses interacting gravitationally. Discuss both the geometries and energies of these orbits.

(💣) What is meant by inertial mass versus gravitational mass? (💣) What is the *Principle of Equivalence*?

() Discuss differences between inertial and non-inertial reference frames.

() In a uniformly rotating reference frame, what two fictitious forces appear? () Where is each one strongest and weakest?

() What happens to the energy in a damped harmonic oscillator that has been set into motion? Discuss both kinetic and potential energies, their relationship to one another, and how they change over time.

State Newton's Shell Theorem.

State Chasles' Theorem.

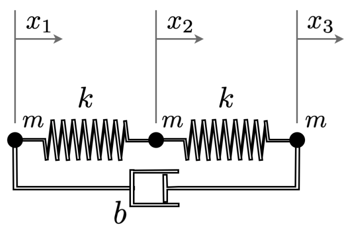
State the Work-Energy Theorem? When does it not apply?

Write Newton's Laws of Motion and discuss their relationship to inertial motion.

Long Answer Question

This is a variant of a question from your last problem set. The process to solve it is exactly the same as on the homework, but with the left and right masses free instead of attached to walls. You will be guided through the process of decoupling the system and will not have to solve it on your own. This will test how well you follow directions and your ability to identify and interpret equations you've encountered multiple times.

Three masses, each mass m, are connected by two springs, each with spring constant k. The first and third mass are connected with a dashpot, (dynamic) damping constant b. The masses are free to move in one dimension, with x_1, x_2, and x_3 representing the displacements from their initial equilibria.



(💣💣💣) Write down the equation of motion for each mass in terms of the natural frequency $\omega_0 = \sqrt{k/m}$ and (kinetic) damping constant $\gamma = b/m$. (*Hint: the next next question may help you get the signs right if you're having trouble.*)

(💣) Explain what is meant by each adjective when we say these equations of motion form a system of **coupled**, **second order**, **linear** differential equations?

Let $x_1(t), x_2(t)$, and $x_3(t)$ be solutions to this coupled system. Show that

$$y_0 = x_1 + x_2 + x_3$$
$$y_1 = x_1 - 2x_2 + x_3$$
$$y_2 = x_1 - x_3$$

constitute a simpler, uncoupled system. Just like the homework, do this by taking time derivatives of each one, plug in the original system of equations of motion, and simplify. () Work carefully to show that y_0 is a particle with no forces acting on it, y_1 is a harmonic oscillator, and y_2 is a damped harmonic oscillator.

() What are the natural frequencies of these oscillators?

() What is the effective frequency of the damped harmonic oscillator?

() Why is y_0 a free particle?

Consider the initial conditions

$$\begin{aligned} x_1(0) &= x_2(0) = x_3(0) \\ \dot{x}_1(0) &= \dot{x}_3(0) = 0 \\ \dot{x}_2(0) &= v_0 \end{aligned}$$

() How would you prepare the system in order for it to have these initial conditions? I'm looking for an operational description (I would preheat the oven to 375^o, then place the cake into it... etc. No math.)

() Describe qualitatively the resulting motion of the system, and draw a picture of it. How do the masses move after a long amount of time has passed? *Hint: think about the asymptotic behavior of the y's and interpret in terms of the x's.*