Autumn 2025

*Professor Sowinski*

# Physics 141: Exam III

University of Rochester

*Department of Physics & Astronomy*

## Instructions

There are 4 questions, you must do all of them.
You have 80 min to catch 16 ☄'s. That's 5 min per ☄.
Show your thinking - partial credit may be earned everywhere.
That means use full sentences as well as mathematics.
Write legibly with a font size greater than 10.
Spherical carbohydrates covered
in glucose will soon appear.
May the Force be with you.

*On my honor and reputation as a scholar of Natural Science,*
*I will neither give nor receive unauthorized help on this exam,*
*and hereby vow that all work will be my own.*

Signature ______________________________

| | | |
|---|---|---|
| P1 | ☄ | |
| | ☄ | |
| | ☄ | |
| P2 | ☄ | |
| | ☄☄ | |
| P3 | ☄ | |
| | ☄☄ | |
| | ☄☄ | |
| P4 | ☄☄☄☄☄ | |

| | | | |
|---|---|---|---|
| $G = 6.674 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$ | $c = 2.998 \times 10^8 \frac{\text{m}}{\text{s}}$ | $h = 6.626 \times 10^{-34} \text{J} \cdot \text{s}$ | $k_B = 1.381 \times 10^{-23} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K}}$ |
| $M_\oplus = 5.972 \times 10^{24}\text{kg}$ | $M_\odot = 1.989 \times 10^{30}\text{kg}$ | $R_\oplus = 6.371 \times 10^6\text{m}$ | $R_\odot = 6.957 \times 10^8\text{m}$ |
| $AU = 1.496 \times 10^{11}\text{m}$ | $M_☾ = 7.348 \times 10^{22}\text{kg}$ | $R_☾ = 1737\text{km}$ | $r_{☾-\oplus} = 3.844 \times 10^5\text{km}$ |

| | Cartesian Coordinates | Polar Coordinates | Cartesian | ⇔ Polar |
|---|---|---|---|---|
| | $\boldsymbol{r} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$ | $\boldsymbol{r} = r\hat{\boldsymbol{r}}$ | $x = r\cos\theta$ | $r = \sqrt{x^2+y^2}$ |
| $\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = \dot{\boldsymbol{r}}$ | $\dot{\boldsymbol{r}} = \dot{x}\hat{\boldsymbol{\imath}} + \dot{y}\hat{\boldsymbol{\jmath}}$ | $\dot{\boldsymbol{r}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$ | $y = r\sin\theta$ | $\theta = \arctan\frac{y}{x}$ |
| $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \dot{\boldsymbol{v}}$ | $\ddot{\boldsymbol{r}} = \ddot{x}\hat{\boldsymbol{\imath}} + \ddot{y}\hat{\boldsymbol{\jmath}}$ | $\ddot{\boldsymbol{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}$ | | |

| | | | |
|---|---|---|---|
| Newton's 2nd Law : | $\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$ | Momentum : | $\boldsymbol{p} = m\boldsymbol{v}$ |
| Impulse : | $\Delta\boldsymbol{p} = \int \boldsymbol{F}dt$ | Universal Gravitation : | $\boldsymbol{F}_g = -G\frac{m_1 m_2}{\lvert\boldsymbol{r}_2 - \boldsymbol{r}_1\rvert^3}(\boldsymbol{r}_2 - \boldsymbol{r}_1)$ |
| Work : | $W = \int_C \boldsymbol{F}(\boldsymbol{r}) \cdot d\boldsymbol{r}$ | Kinetic Energy: | $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ |
| Work-Energy Theorem: | $W = \Delta K$ | Conservative Force : | $\boldsymbol{F} = -\nabla U$ |
| Spring Potential Energy : | $U_s = \frac{1}{2}m\omega^2\lvert\boldsymbol{r} - \boldsymbol{r}_{eq}\rvert^2$ | Potential Energy : | $U(\boldsymbol{r}_f) - U(\boldsymbol{r}_i) = -\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ |
| Grav. Potential Energy : | $U_g = -G\frac{m_1 m_2}{\lvert\boldsymbol{r}_2 - \boldsymbol{r}_1\rvert}$ | Mass of object : | $M = \int_\Omega \rho(\boldsymbol{r})dV$ |
| Center of mass: | $\boldsymbol{R} = \frac{1}{M}\sum_i m_i\boldsymbol{r}_i$ or $\frac{1}{M}\int_\Omega \rho(\boldsymbol{r})\boldsymbol{r}dV$ | | |
| Angular Momentum: | $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} = I\boldsymbol{\omega}$ | Torque: | $\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = I\boldsymbol{\alpha}$ |
| Moment of Inertia: | $I = \sum_i m_i(r_i^\perp)^2$ | | |

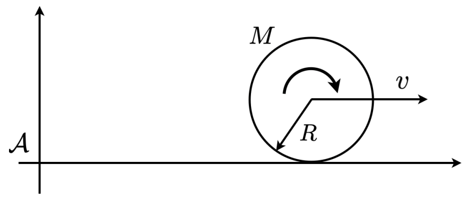
$$\boldsymbol{A} \cdot \boldsymbol{B} = \lvert\boldsymbol{A}\rvert\lvert\boldsymbol{B}\rvert\cos\theta \quad \lvert\boldsymbol{A} \times \boldsymbol{B}\rvert = \lvert\boldsymbol{A}\rvert\lvert\boldsymbol{B}\rvert\sin\theta \quad \boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B})$$

Expanding $\delta x$ around $x$ $\quad f(x + \delta x) = f(x) + f'(x)\delta x + \frac{1}{2!}f''(x)\delta x^2 + \cdots$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - \cdots \qquad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \qquad \ln\lvert 1+x\rvert = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots \qquad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \qquad \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

**Problem 1**

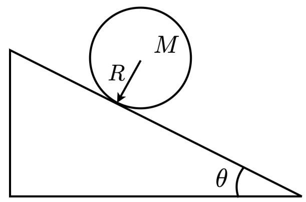



A uniform **cylinder** of mass $M$ and radius $R$ rolls horizontally without slipping at a speed $v$.
( ) What is the cylinder's **spin** angular momentum?

( ) What is the cylinder's **total** angular momentum relative to the origin, point $\mathcal{A}$? It may help you to draw the position vector pointing from the origin to the cylinder's center of mass.

( ) What does Chasles' Theorem tell us we can do when describing the motion of a rigid body?

**Problem 2**

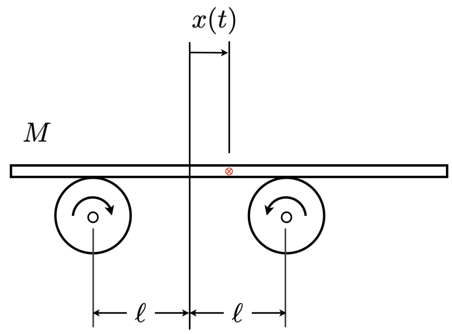



A uniform **sphere** of mass $M$ and radius $R$ is placed at rest on a fixed incline in a gravitational field of strength $g$ directed downward.

(💣) If there is no friction between sphere and incline, find the acceleration of the sphere. Drawing a free-body diagram will help.

(💣💣) With friction the sphere rolls without slipping. Find the acceleration of the sphere when there is friction. Make sure to compare your answer to the previous one to check if it makes sense.

**Problem 3**




Two wheels of equal radii are fixed in place a distance $2\ell$ apart and made to spin towards one another at the same angular speed. There is a gravitational field of strength $g$ acting in the vertical direction. A plank of mass $M$ is placed on the wheels such that its center of mass ($\otimes$ in the figure) is a distance $x < \ell$ from the center of the two wheels. The coefficient of friction between plank and the left wheel is $\mu_L$, and the right wheel $\mu_R$. ( ) Draw a free body diagram for the plank.

( ) Write down Newton's second law for both translational and rotational motion of the plank. i.e. sum of forces, sum of torques.

( ) Combine them to get a differential equation for $x(t)$. Where is the equilibrium position of the plank? Your answers must only include the given variables. Describe its motion quantitatively.

**Problem 4**

() For this question you need to write a short essay response to this prompt.

*In the* $19^{th}$ *century the English physicist, surveyor, and inventor Henry Kater devised an ingenious way to measure the Earth's gravitational field.*