The Trials and Tribulations of Teleportation

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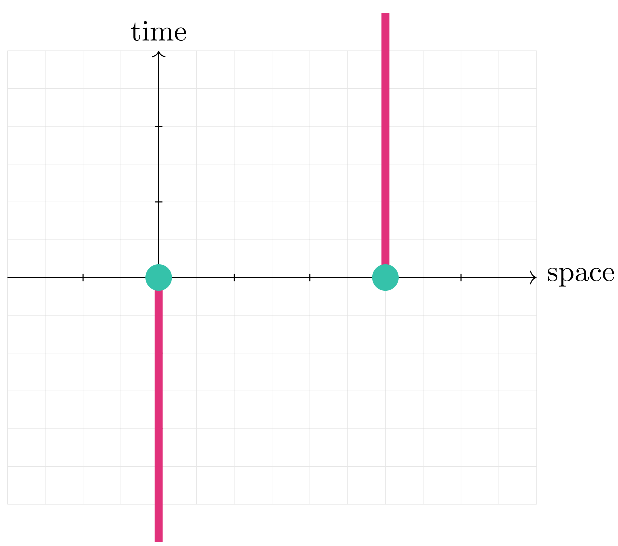
1 Relativity

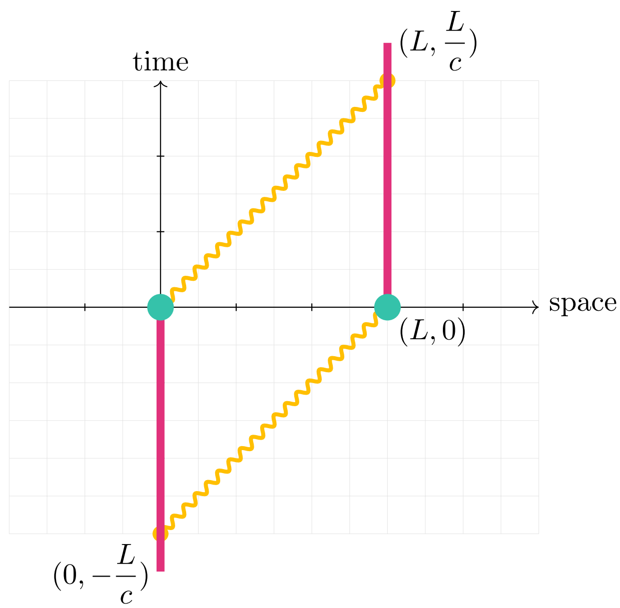
1.1 Teleportation is simple, right?

One moment you're here, and then in the next you're over there. Easy peasy; we've solved the problem. We can even draw a picture of how it. Let space be represented by a horizontal axis, and time a vertical one. Points on this *spacetime* plot are labeled with (x, t) — events occurring at the spatial point x and temporal moment t. Teleportation looks like this:



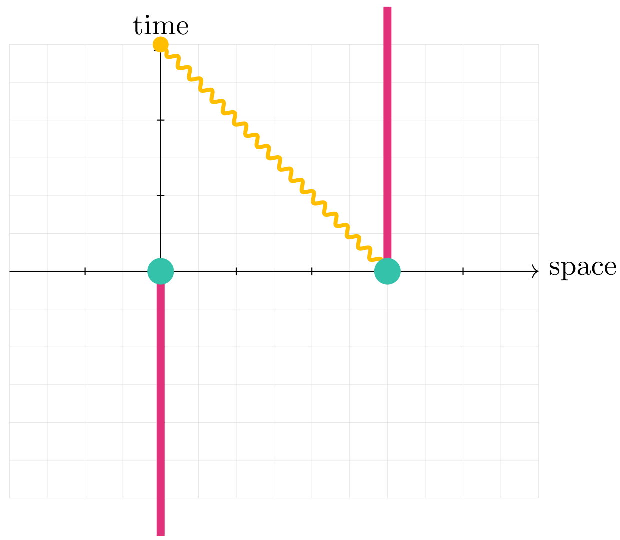
Your history is drawn in pink. You sit at $x = 0$ until your clock ticks up to $t = 0$, at which point you are whisked off to a new position, $x = L$: Awesome. You teleported.

What would you see if this happened? Remember that the speed of light is not infinite, so it takes a bit of time for light to get from one place to another. You can forget all the stuff you know about relativity for now, except that. When you appear at your teleportation destination and look back at your origin, you will see yourself!



In fact you will see yourself as you were a time L/c in the past. You will stare at yourself, sitting there until L/c time has passed, at which point your image disappears.

You might consider waving at your past self. As a physicist who might want to induce a catastrophic temporal paradox, you can tell them to NOT teleport. Alas, your nefarious schemes will not work.

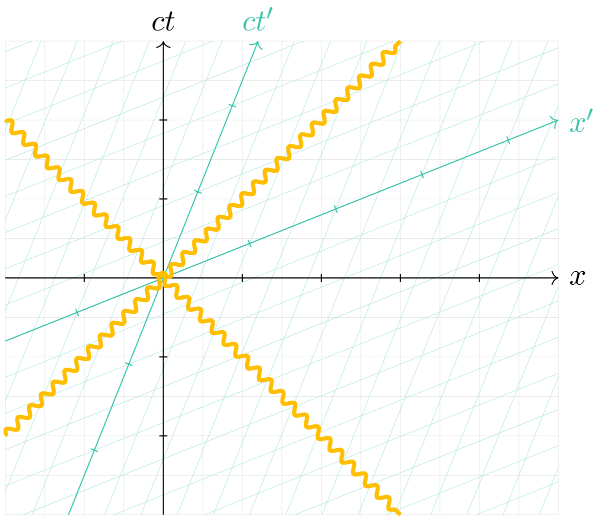


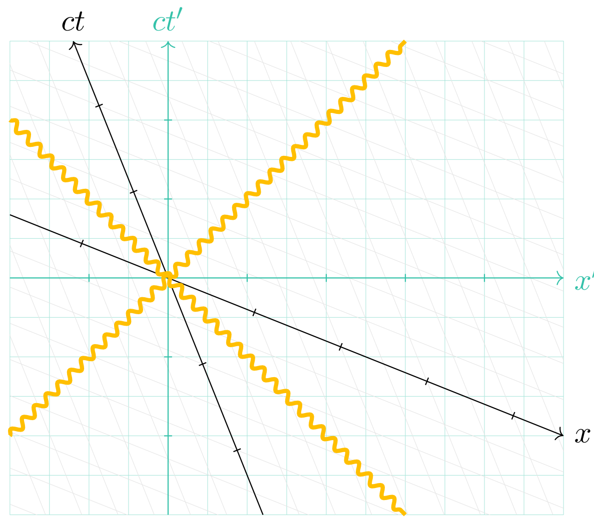
Even if you start signaling the moment you've finished teleporting, the light you create won't reach your past self.

Or could it?

1.2 Watching someone else teleport

It's time to remember a little bit about the geometry of spacetime. Forget about teleportation, and let's recall some basics. Even though all the pictures we've been looking at have a single worldline in them — yours — there's actually someone else in them too — me! The space and time axes that I keep drawing are mine. You and I are not moving relative to each other, so your worldline has lined up with my axes. But, what if there was relative motion between us? Since spacetime is Lorentzian, my space and time coordinates would not be the same as your space and time coordinates. In the left figure is what your (ct, x) and my (ct', x') axes look like to you, and in the right what they would look like for me. The lightcones are there to remind us of the first postulate of relativity: even though we have different coordinate systems, we measure the speed of light to be the same.



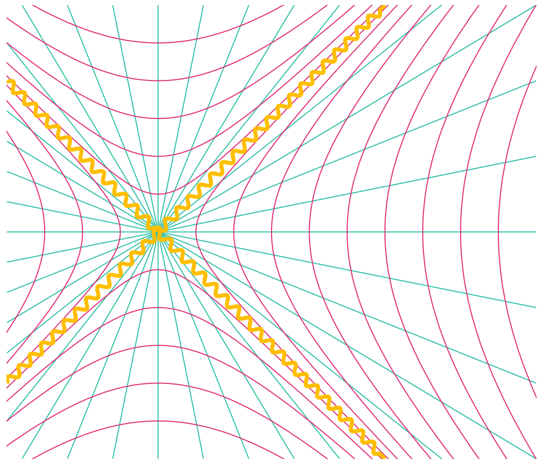


They are related by a Lorentz transformation, specifically a *boost* which rotates space and time into one another.

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} \qquad \text{where} \qquad \eta = \frac{1}{2}\ln\frac{c+v}{c-v} \quad \text{is the } \textit{rapidity} \text{ factor.} \tag{1}$$

Note where the tick marks are on each axis.

If we changed the rapidity between us, those tick marks will sweep out hyperbolas. Below we see the spacelike and timelike hyperbolas corresponding to constant duration and distance, respectively, for all observers, irrespective of their rapidity relative to you, at the origin.



Ok, neat, that's a lot of curves. So what? Well, what we have in front of us is the key to proving something interesting about teleportation. Namely, teleportation and time travel are the same thing.

What?

You heard me. There's no difference between a teleporter — a device that makes an object jump through space — and a time machine — a device that makes an object jump through time. This should feel right, after all the Lorentzian nature of spacetime means that different observers will mix their definitions of time and space into one another. Let's build some geometric intuition to get a more quantitative understanding of this claim.

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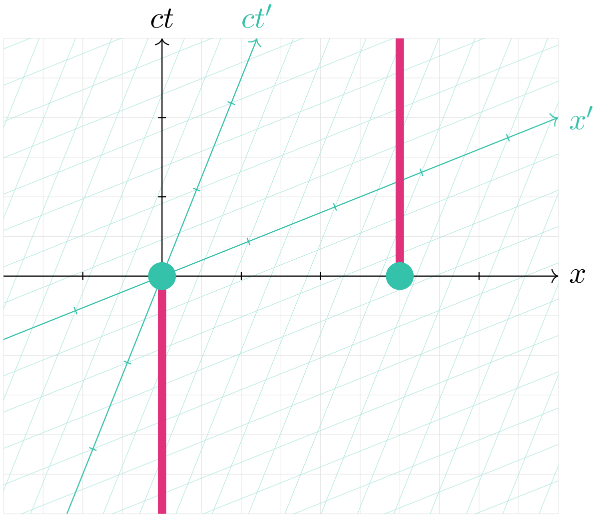
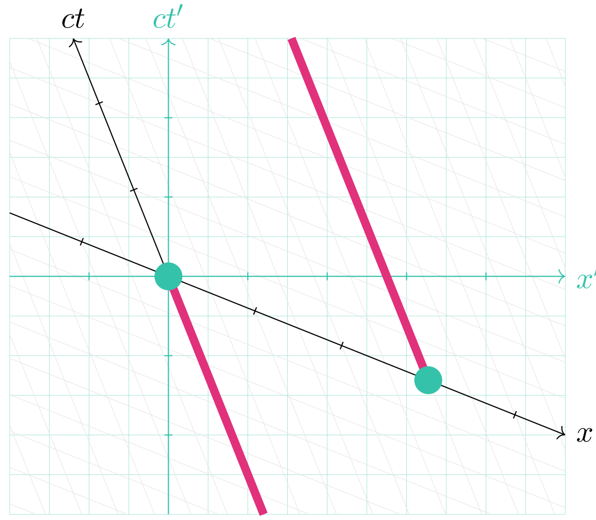
Let's go pack to the very first figure in the set of notes, the one in which we drew teleportation. That's what happened in your reference frame, and it is shown down below in the left. My coordinate system has been superimposed to remind you what it looks like in your frame. Now let's jump into my perspective by boosting into my frame. Your coordinate system is now moving relative to my own, shown in below to the right. Notice that in my frame you start to teleport AFTER you finish teleporting!



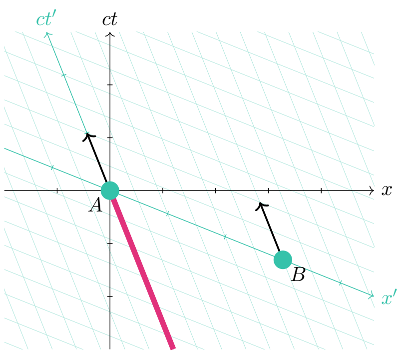


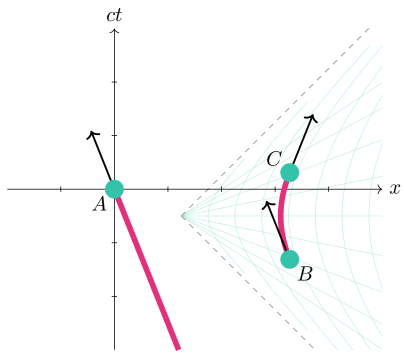
If you've studied relativity before, this should not be too much of a surprise. The start and end of teleportation are spacelike separated events. Spacelike separated events are not causally connected, so there is no unique notion of before and after between them.

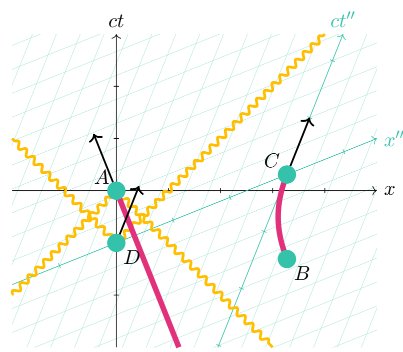
Even though time travel occurred from one point of view, the universe is still protected from any paradoxes. The light cone coming from the destination event never intersects the worldline containing the starting point. To induce a spacetime continuum catastrophic paradox, one requires a rocket.

1.3 Teleportation is Time Travel

If you want to cause a paradox with a teleporter, you're going to need a rocket. Below, the left panel is just our standard picture of teleportation to the right from event A to event B. Sure there's time travel, but the innocuous kind. As soon as you exit the teleport destination you turn your rockets on in the direction you're moving. You accelerate constantly from event B to event C; according to me you're now moving in the opposite direction. The Rindler wedge is in the panel for you nerds who have read my other notes. At event C you turn on your teleporter again, this time teleporting to the left. This is shown below in the rightmost panel, where you end up at point D.

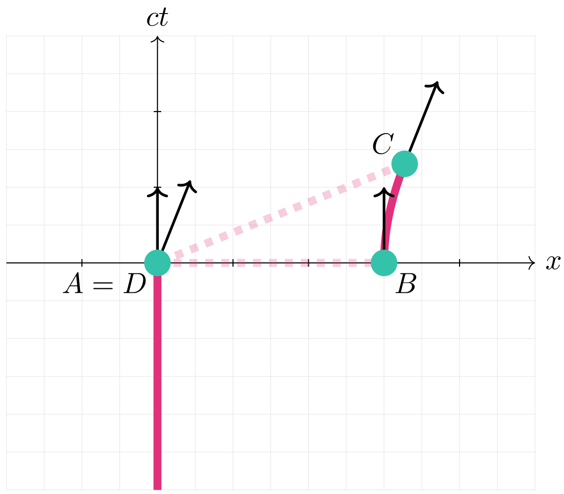






There's the kicker. Event A is in the future lightcone of event D, as D is in the past lightcone of A. After reaching D, you can influence and part of the worldline laying in the intersection of those two lightcones. You can, for instance, tell your past self to not teleport. If you know you're an obstinate individual and that your past self will ignore any such message, you can just break the teleporter instead. A spacetime paradox will ensue. If you teleported then you didn't teleport, but if you didn't teleport then you did. I get headaches thinking about it.

Even though this was done from the perspective of the non-moving frame, the result holds in any reference frame. Let's work through a problem in your frame, assuming the teleporter is a small device that you can hold on to. Let's say that the teleportion device can only whisk you a distance L (in your own restframe, because you're holding the device). What proper acceleration must you undergo, and for how long (propertime), in order for your second teleport to bring you back to the where and when you first teleported? See the figure below.



Events A and D are at $(0,0)$, while point B is at $(L,0)$. The hyperbola of constant acceleration at B has

$$ct(\tau) = \frac{c^2}{a}\sinh\frac{a\tau}{c} \tag{2}$$

$$x(\tau) = L - \frac{c^2}{a} + \frac{c^2}{a}\cosh\frac{a\tau}{c}. \tag{3}$$

At C you have a rapidity relative to the original coordinates of $\eta = a\tau/c$. The coordinate system moving at this rate has

$$\begin{bmatrix} ct_D \\ x_D \end{bmatrix} = \underbrace{\begin{bmatrix} \cosh\eta & -\sinh\eta \\ -\sinh\eta & \cosh\eta \end{bmatrix}}_{\text{back to the original frame}} \left(\underbrace{\begin{bmatrix} \cosh\eta & \sinh\eta \\ \sinh\eta & \cosh\eta \end{bmatrix} \begin{bmatrix} \frac{c^2}{a}\sinh\eta \\ L - \frac{c^2}{a}(1-\cosh\eta) \end{bmatrix}}_{\text{into the instantaneous rest frame}} - \underbrace{\begin{bmatrix} 0 \\ L \end{bmatrix}}_{\text{teleportation}} \right) \tag{4}$$

Since $ct_A = x_A = 0$, this means (check the algebra!) that L must be equal to the Rindler horizon, so that $La = c^2$. This leaves the proper time spent accelerating as a free parameter. It determines the velocity you have upon returning back, $\beta = \tanh\eta = \tanh\frac{c\tau}{L}$. So, if you ever see an object instantaneously change its speed, double check if it also aged discontinuously.

2 Teleporting a pair of particles

There's the question of how the teleporter works. Is there a single reference frame associated with the teleporter? Or does the teleport happen within the reference frame of each object being teleported?

2.1 Teleporting a pair of particles

We've seen that teleporting twice can be used to create strange time loops. Temporal paradoxes aside, there is more weirdness when we consider teleporting an object made of two or more particles. We're going to treat the pair classically for now; don't worry, Quantum shenanigans are in the next section. For now, let's imagine teleporting an obect made of two particles, each of mass m, a distance L. At the moment of teleportation, the pair are a distance $d \ll L$ apart. The particles can jiggle about their equilibria, but let's assume that they have no angular momentum: their motion is essentially one dimensional. In the center of mass frame, at the moment of teleportation, one has speed v and the other $-v$. You can check that their rapidities are η and $-\eta$.